Balanced Label Propagation for Partitioning Massive Graphs

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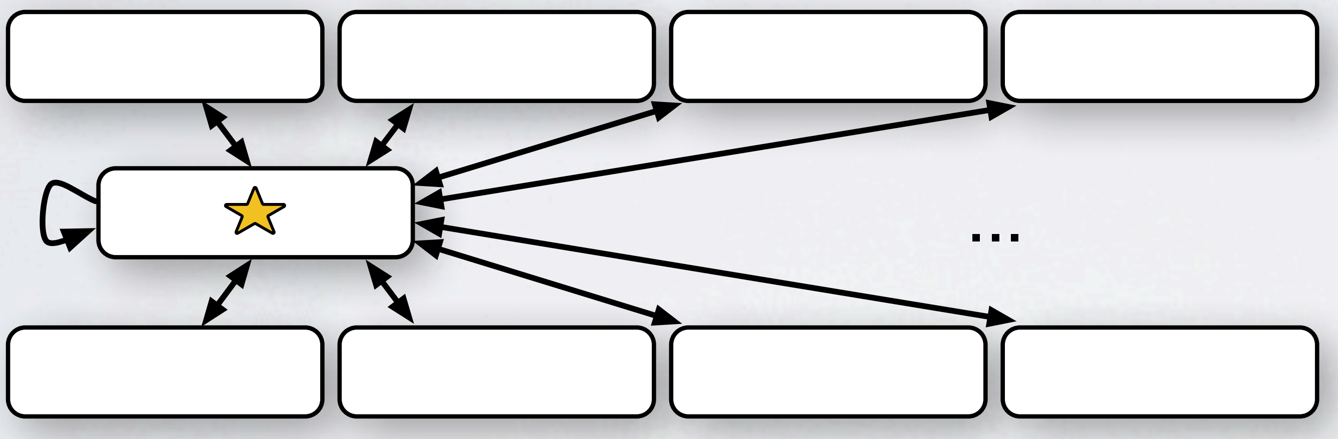
facebook

Goal: partition a really big graph

Motivation: distributed computation

- Distributing graph calculations ('sharding a graph') makes traversal/aggregation very expensive.



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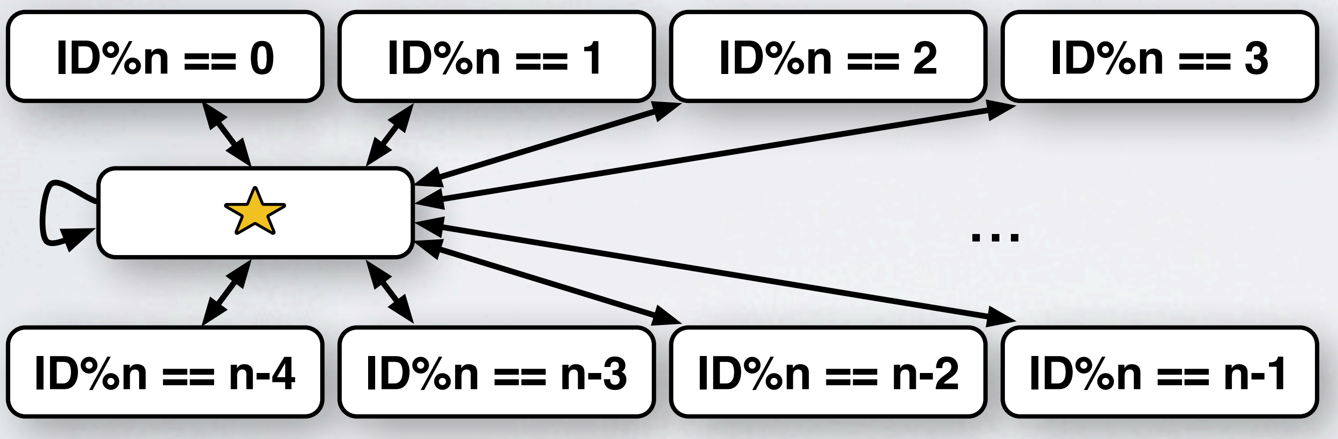

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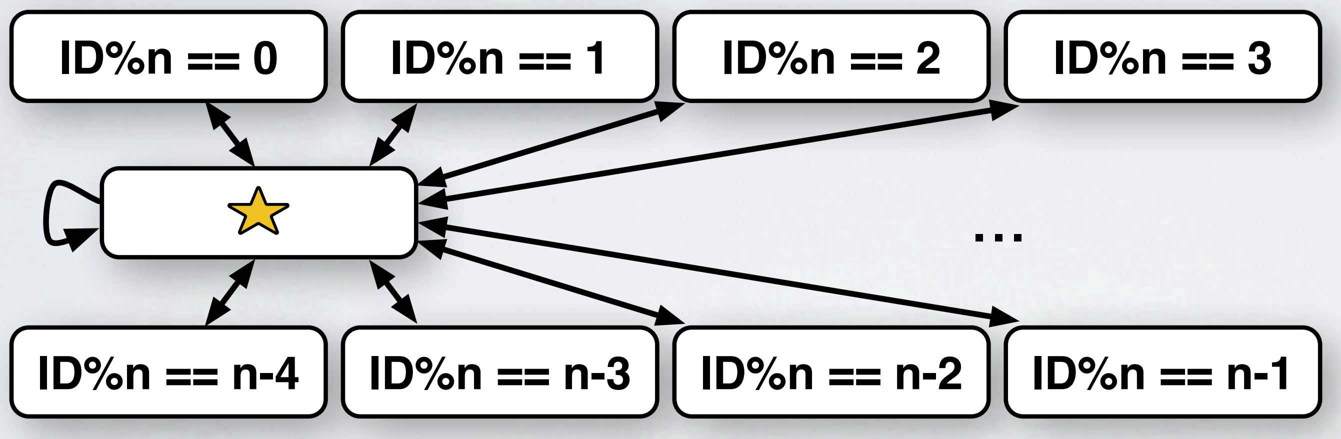

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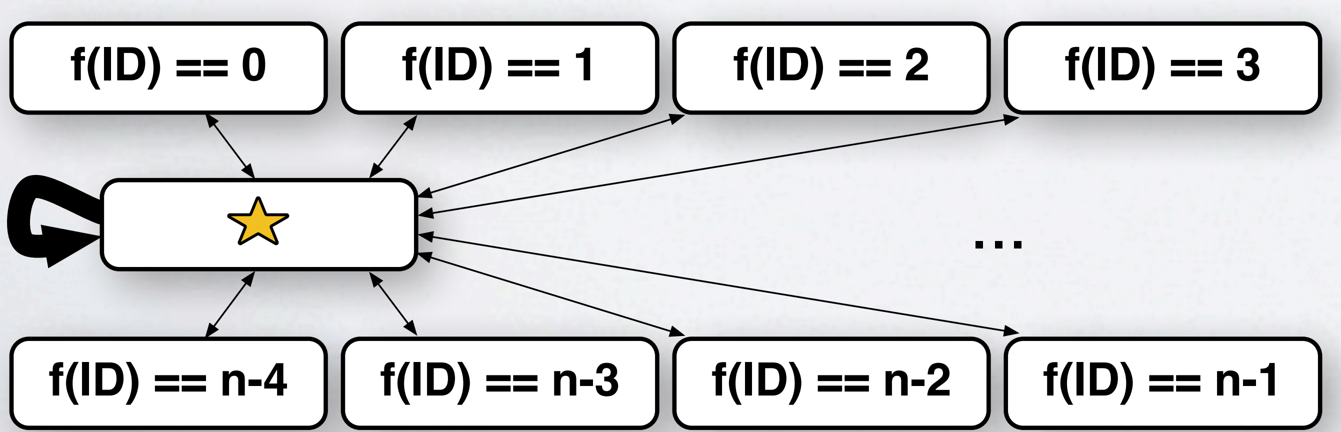
- **Intelligent sharding**: specify a *shard map* **f()** that colocates users with friends



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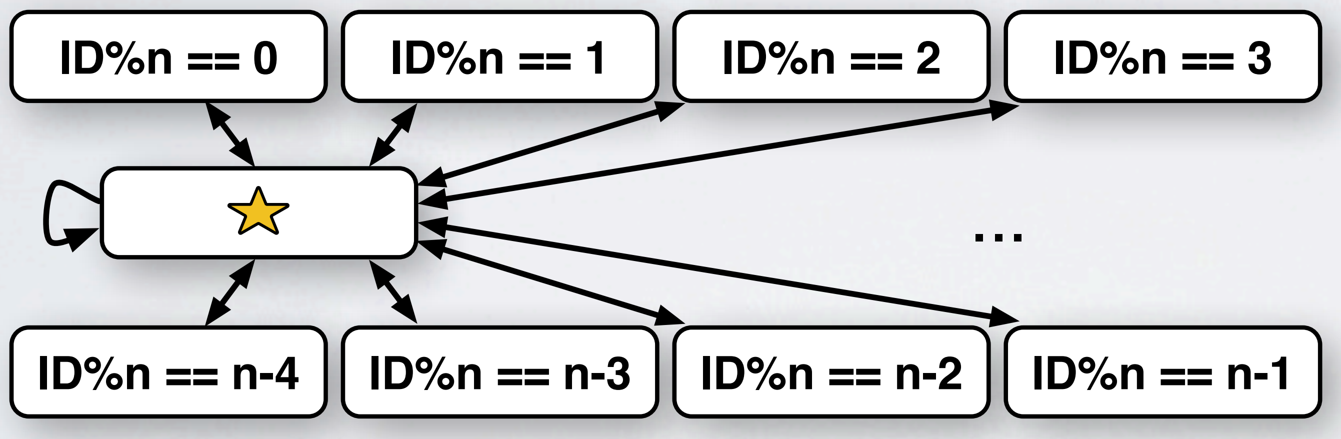

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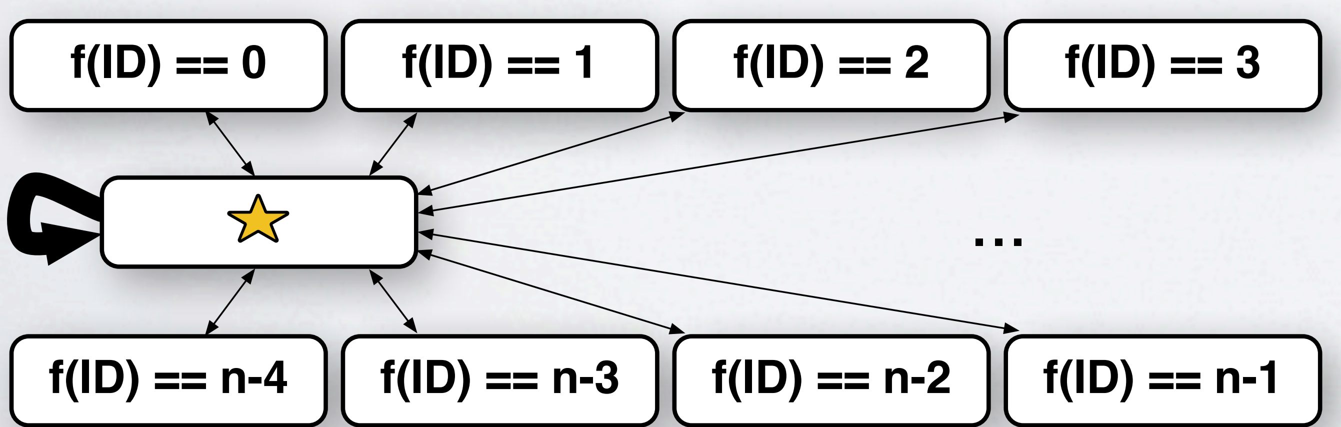

Oh… and the algorithm better be FAST.

Partitioning a really big graph: How?

- Garey, Johnson, Stockmeyer 1976: Minimum bisection is NP-hard
- Karypsis and Kumar 1998: METIS
- Feige and Krautgamer 2000: $O(n^{1/2} \log n)$-factor approximation

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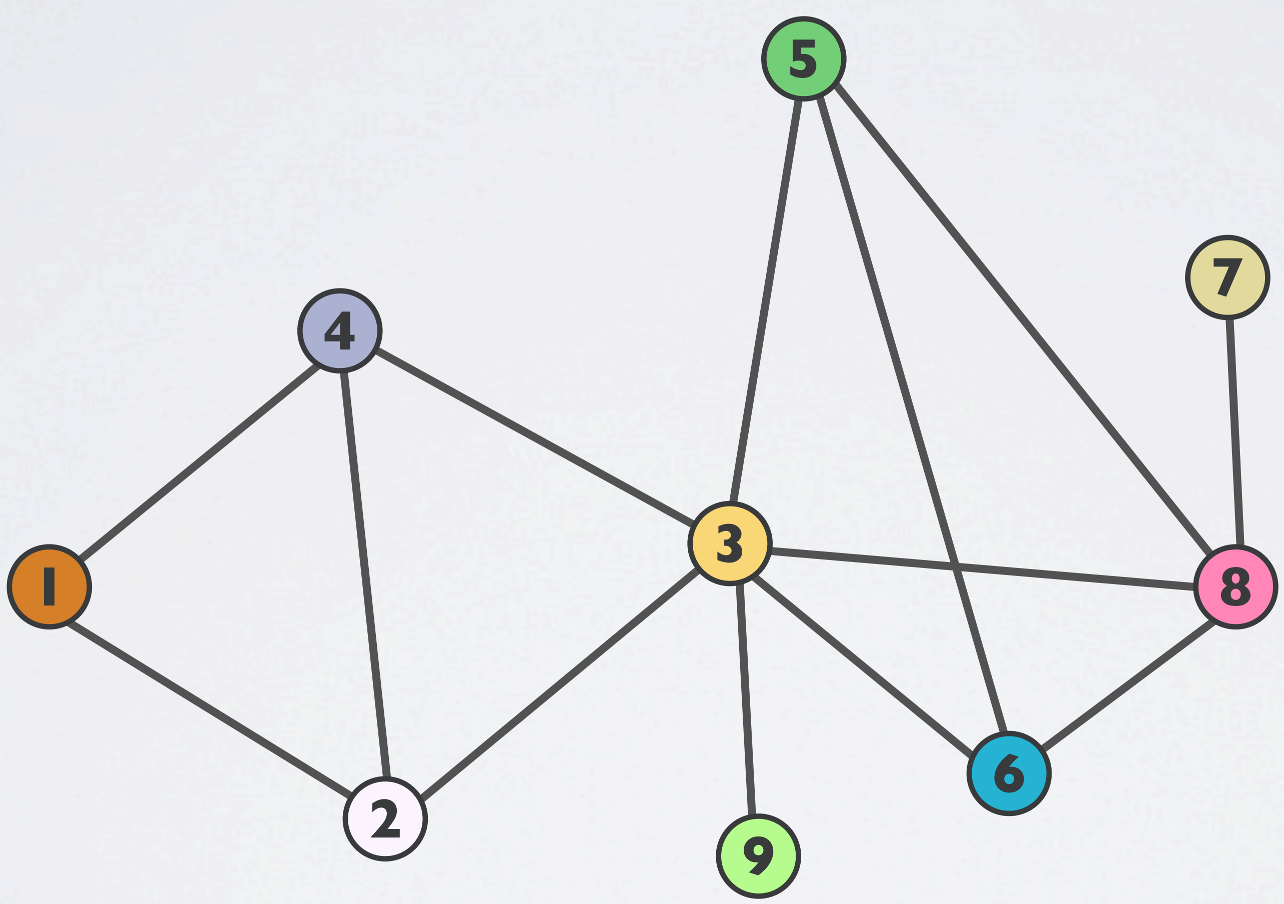
- METIS does not scale to 100B+ edges.
- Need a principled approach, ideally one that can be Hadoop-ified.

Basic idea: Label propagation

- Iteratively move nodes to be with the plurality of their neighbors:



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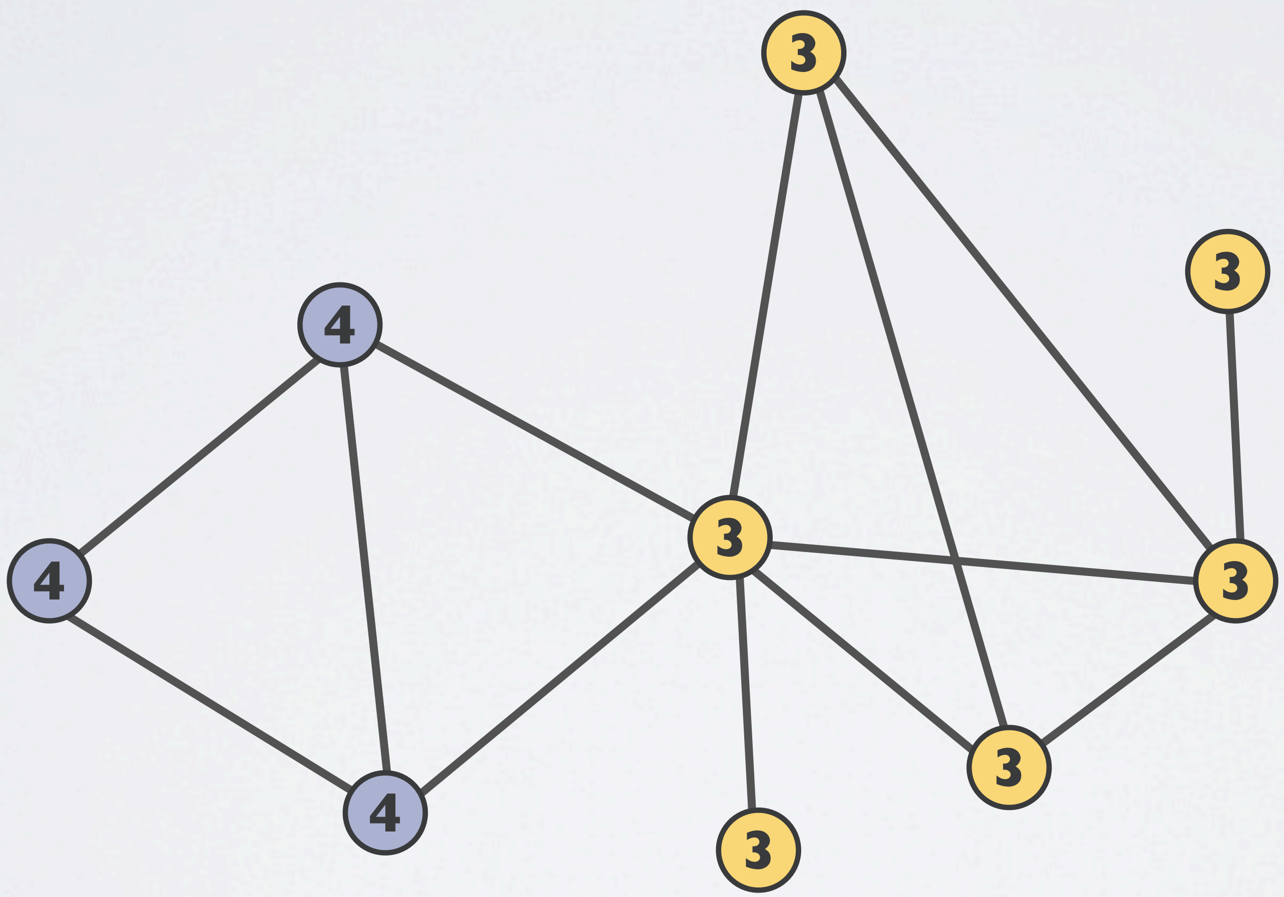

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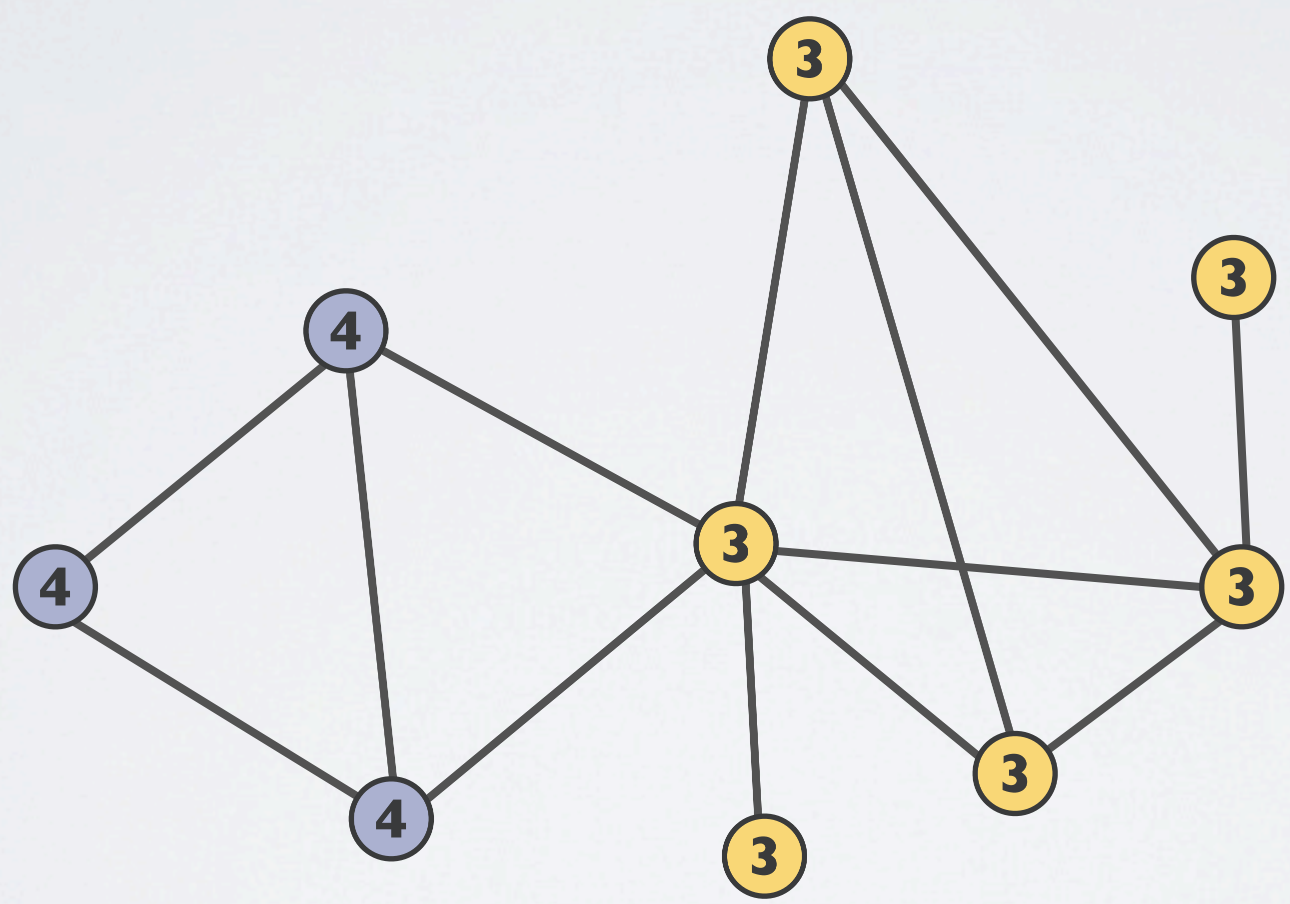

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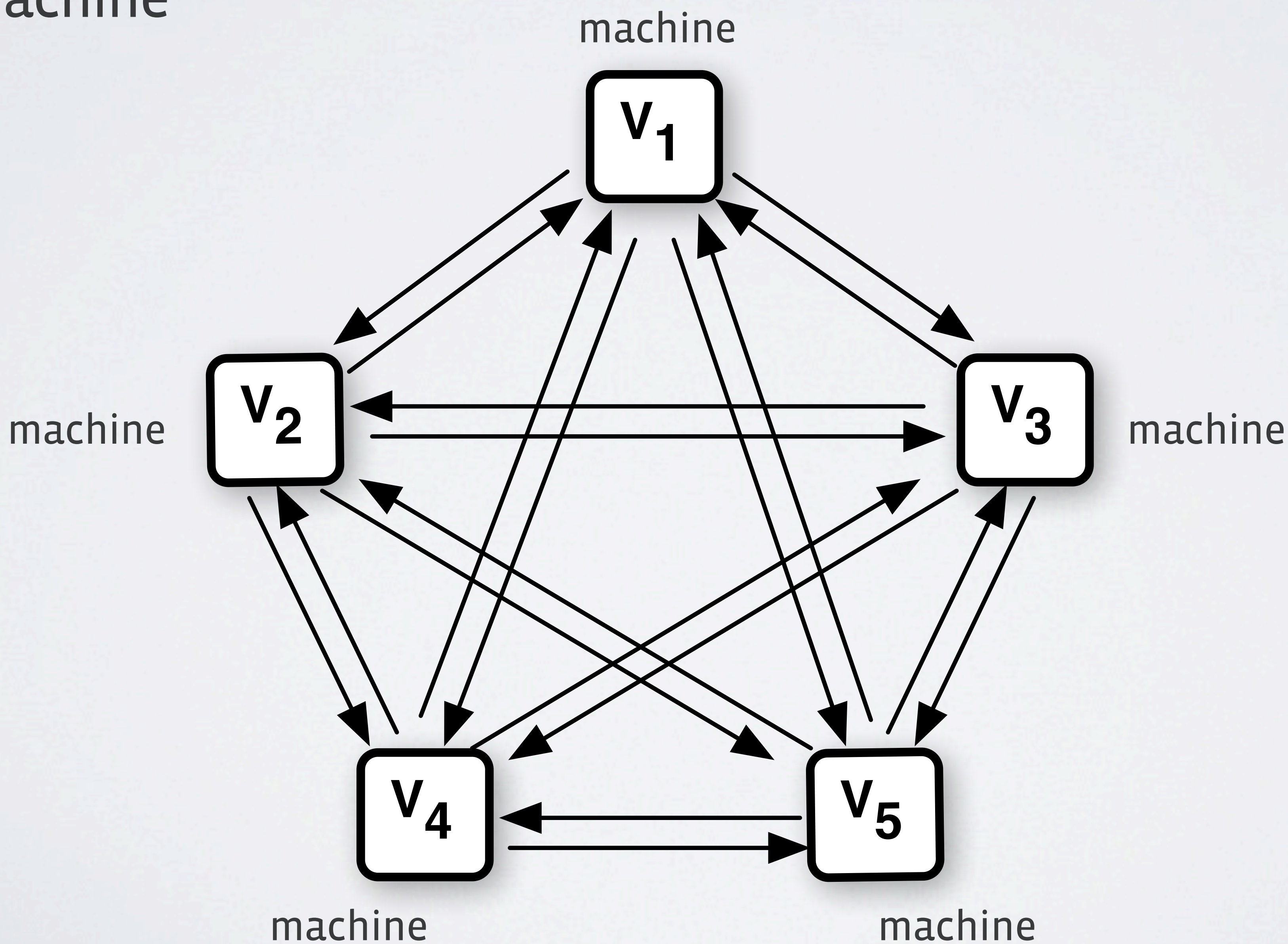

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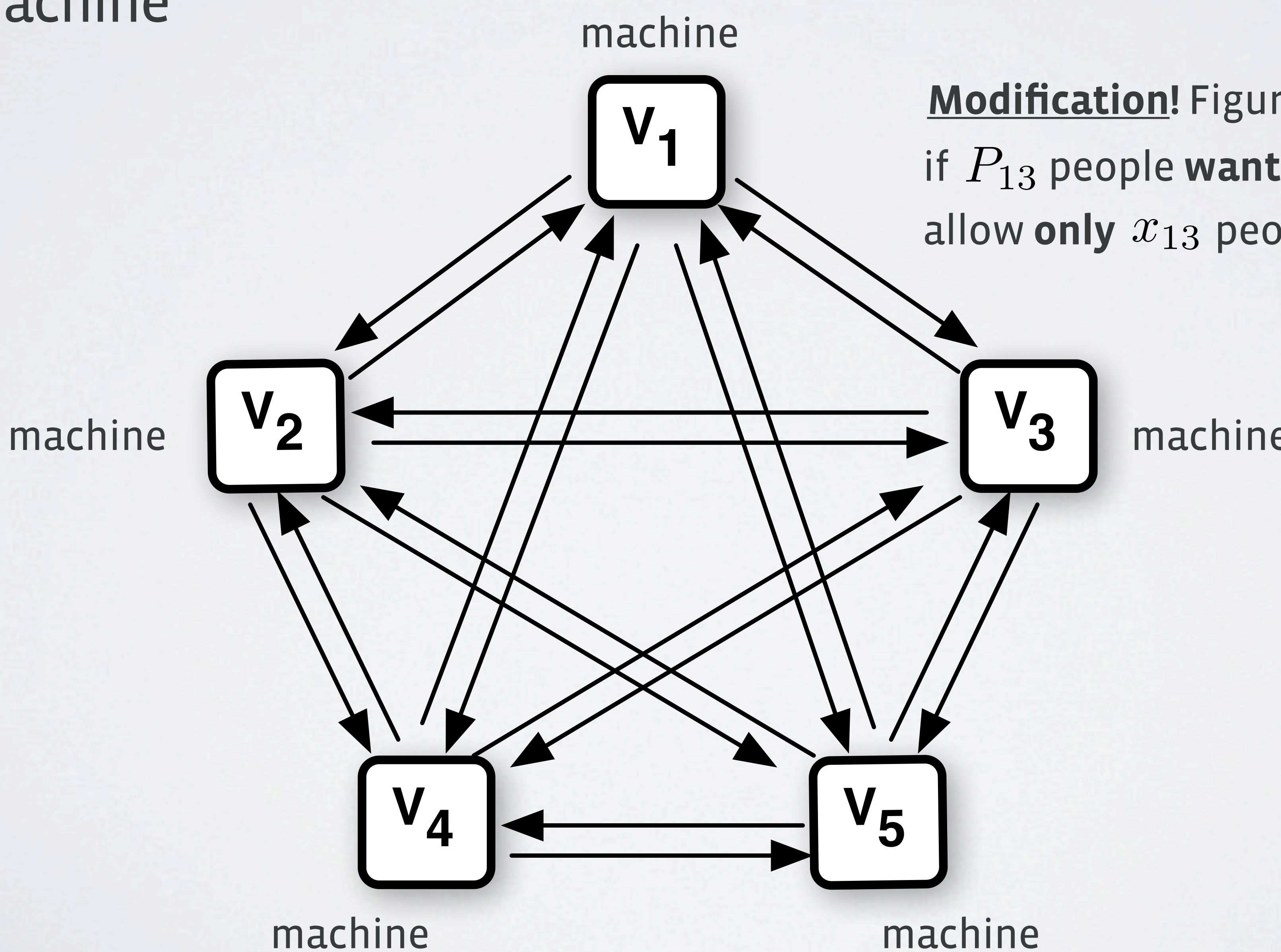

<u>Modification</u>! Figure out who wants to move.

if P_{13} people **want** to move from 1 to 3.

allow **only** x_{13} people move, **s.t. flow balance.**

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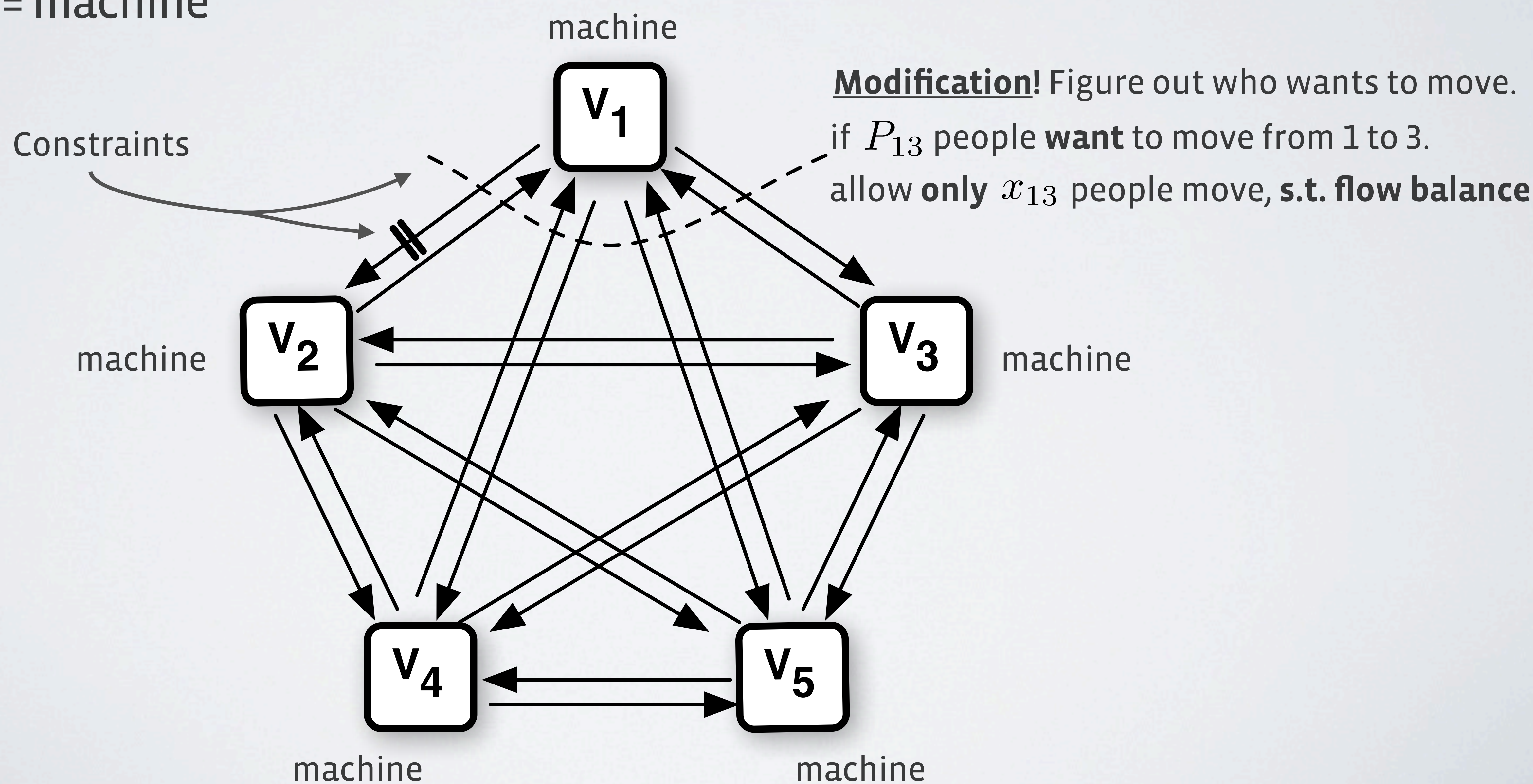

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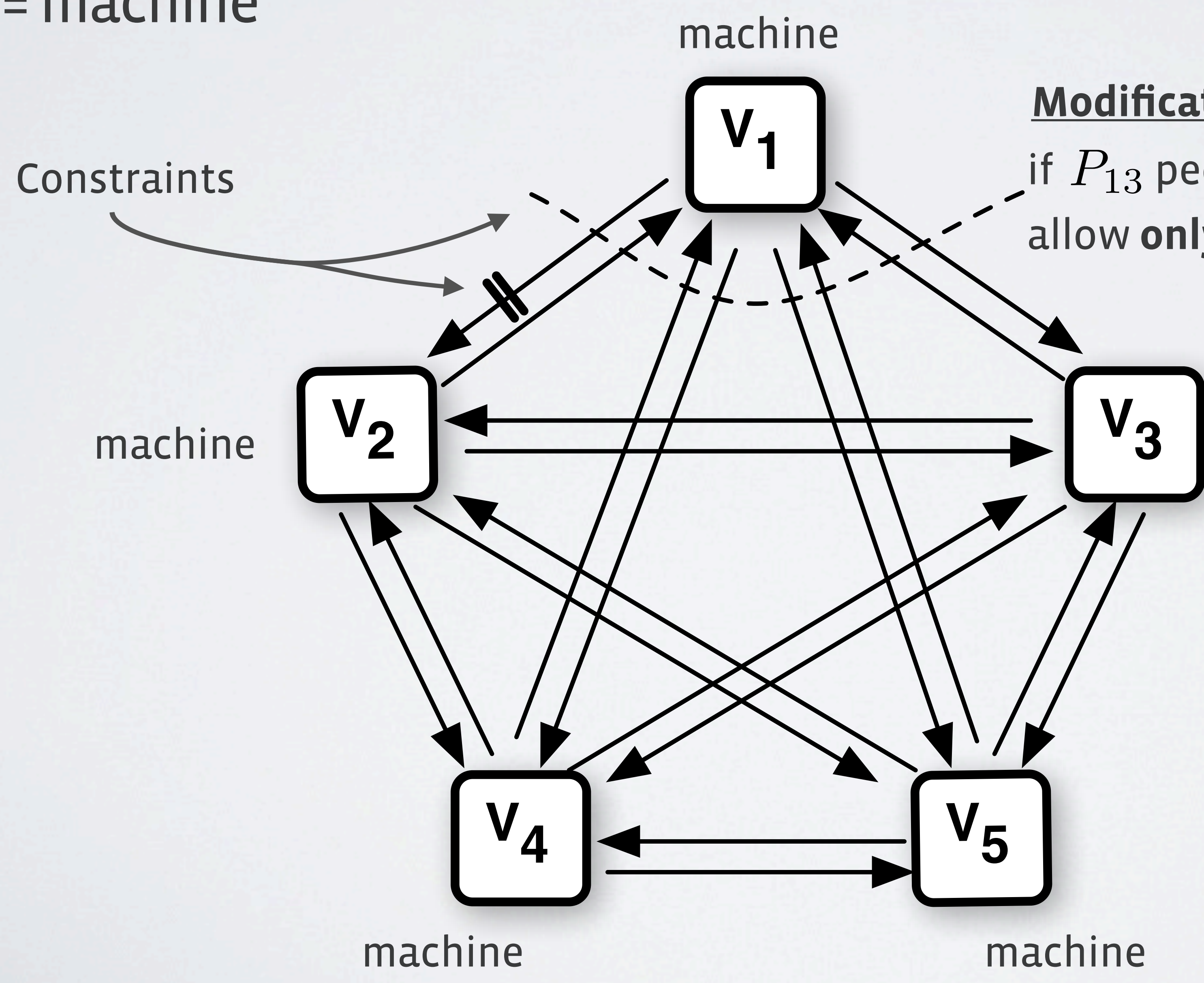

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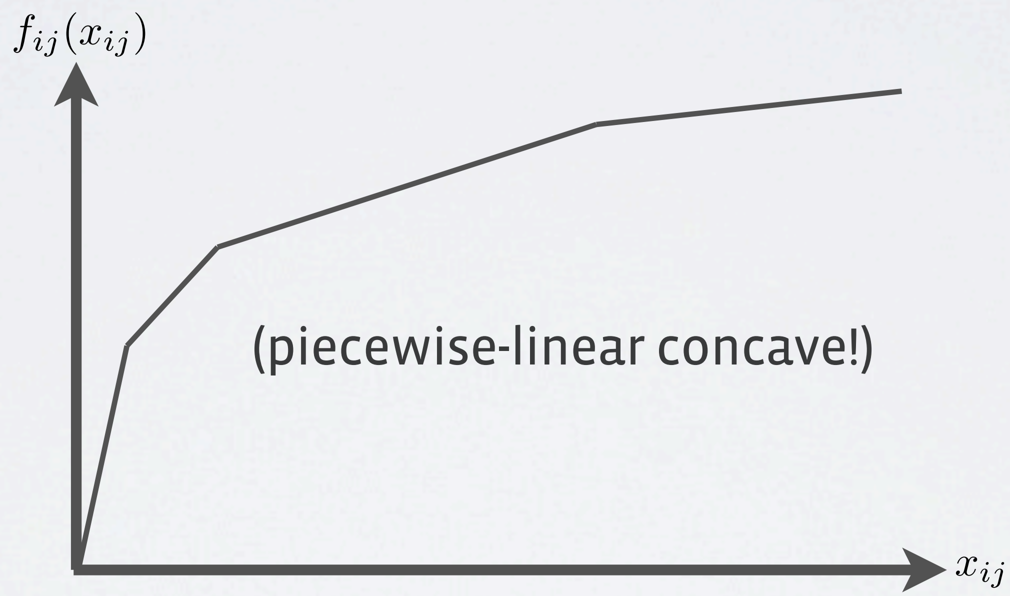
How do we solve for x_{ij}?

Balance via Linear Program

- **Greedily** maximize edge locality with constraints (max/min sizes S_i,T_i):

x_{ij}	Solution: number of people to move from i to j.
$f_{ij}(x)$	Cumulative gain from moving x people **(ordered by co-location gain)**.



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(Maximize the co-location gain of all machine swaps)

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- **Linear Program**: n=78 machines => 12k variables / 400k constraints

$$\max_{X,Z} \sum_{i,j} z_{ij} \qquad \text{s.t.} \begin{cases} S_i - |V_i| & \leq & \sum_{j \neq i}(x_{ij} - x_{ji}) & \leq & T_i - |V_i|, & \forall i \\ 0 & \leq & x_{ij} & \leq & P_{ij}, & \forall i,j \\ & & -a_{ijk}x_{ij} + z_{ij} & \leq & b_{ijk}, & \forall i,j,k \end{cases}$$

Balance via Linear Program

- **Summary of algorithm:**
  - Step 1: Figure out who wants to move
  - Step 2: Solve LP to decide who can move without breaking balance
  - Step 3: Move those people

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Step 2 is the contribution compared to ordinary Label Prop.

What about geography?

Initialization using geography

- Possible to do much better than random with Facebook, using **geography**.

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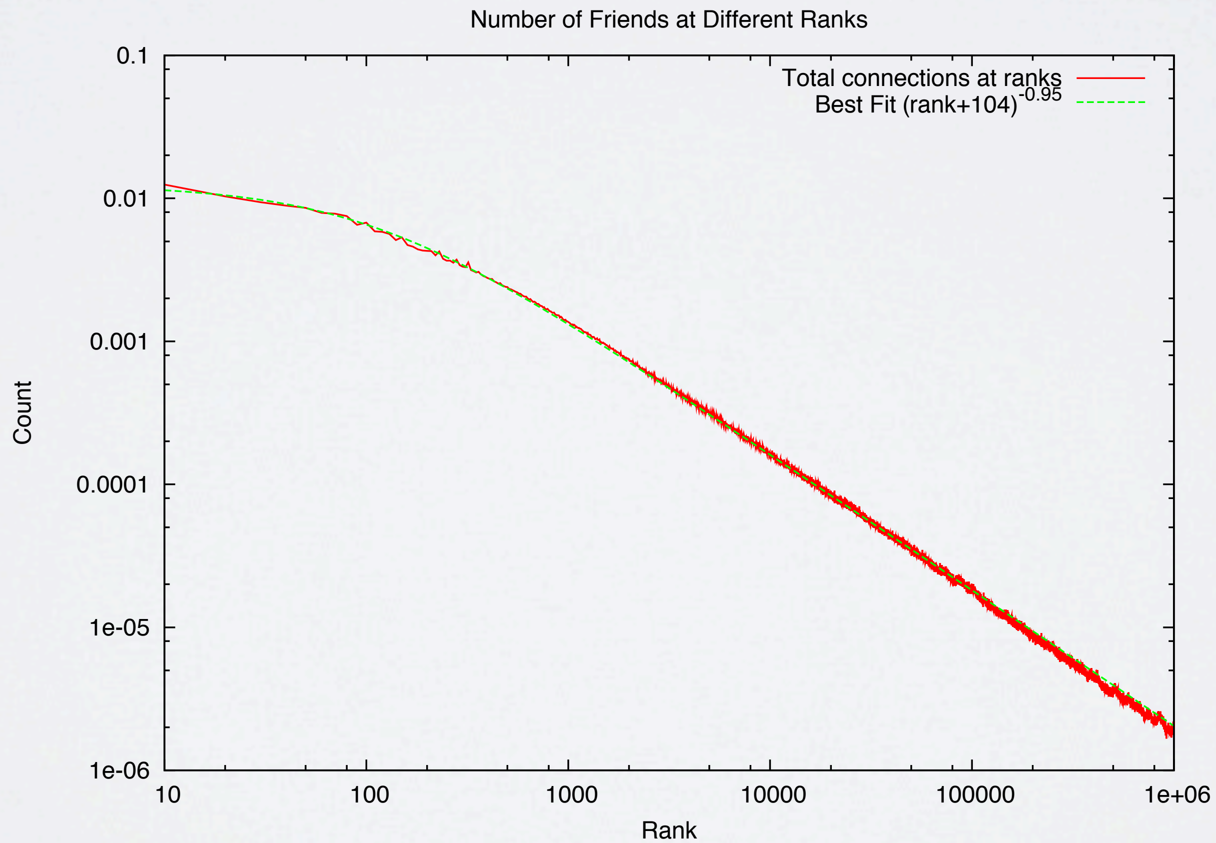
  - **Spatial model** of small-world networks (for routing): Kleinberg 2000
  - **Validation**: Liben-Nowell et al. 2005; Backstrom, Sun, Marlow 2010.
  - Friendship probability as a function of rank-distance:



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Initialization using geography

- IP data reveals geographic location of users:
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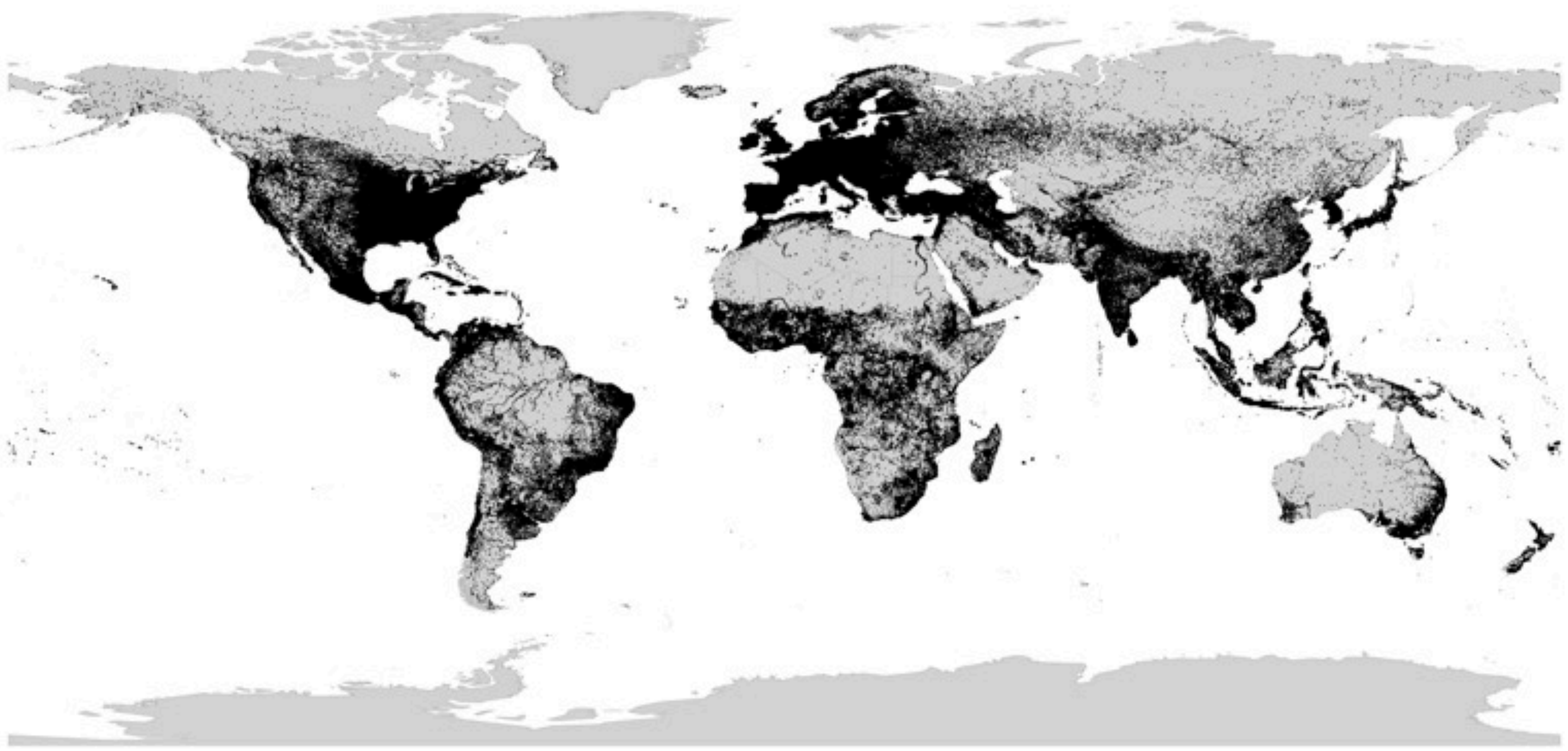

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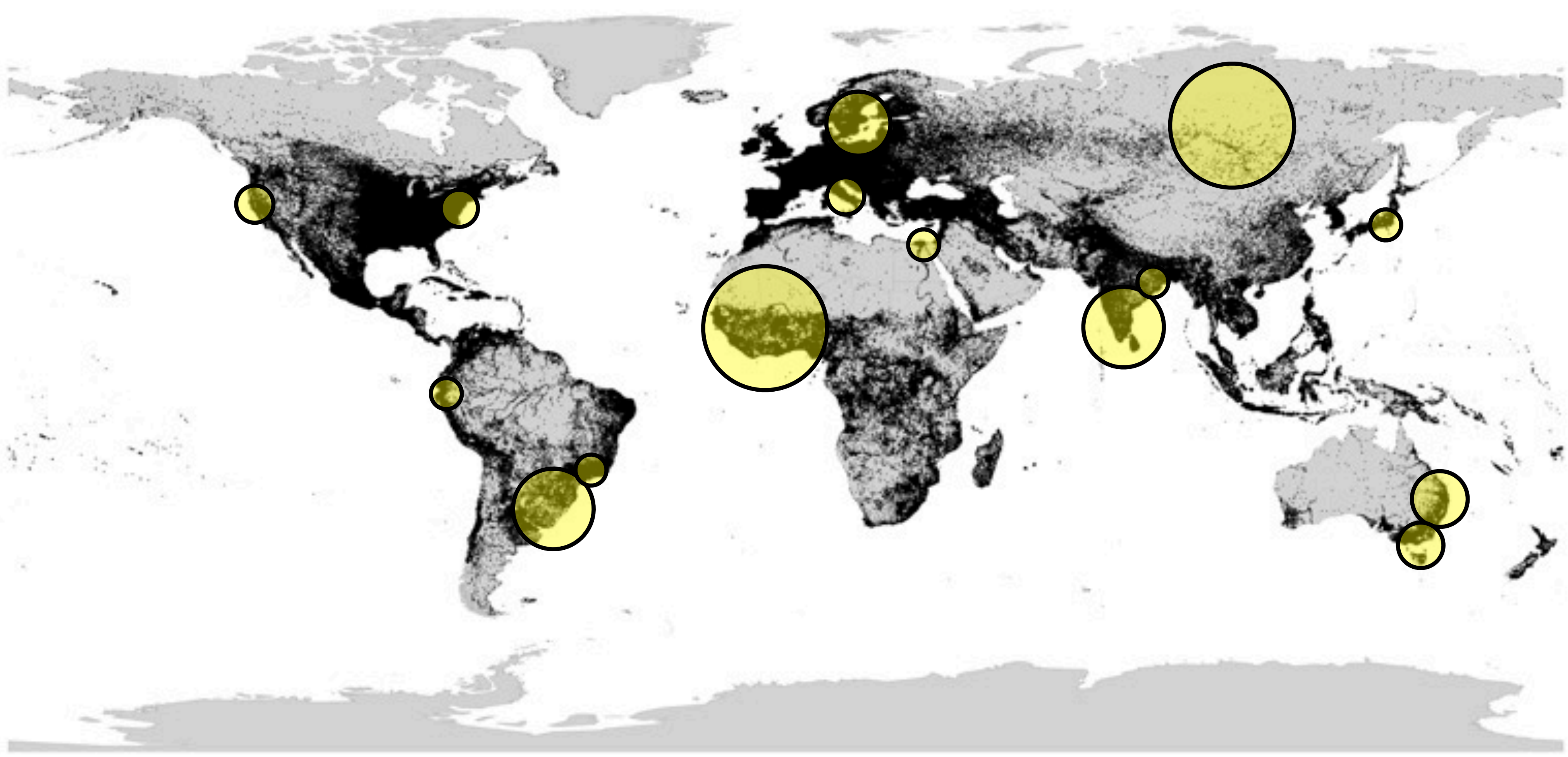

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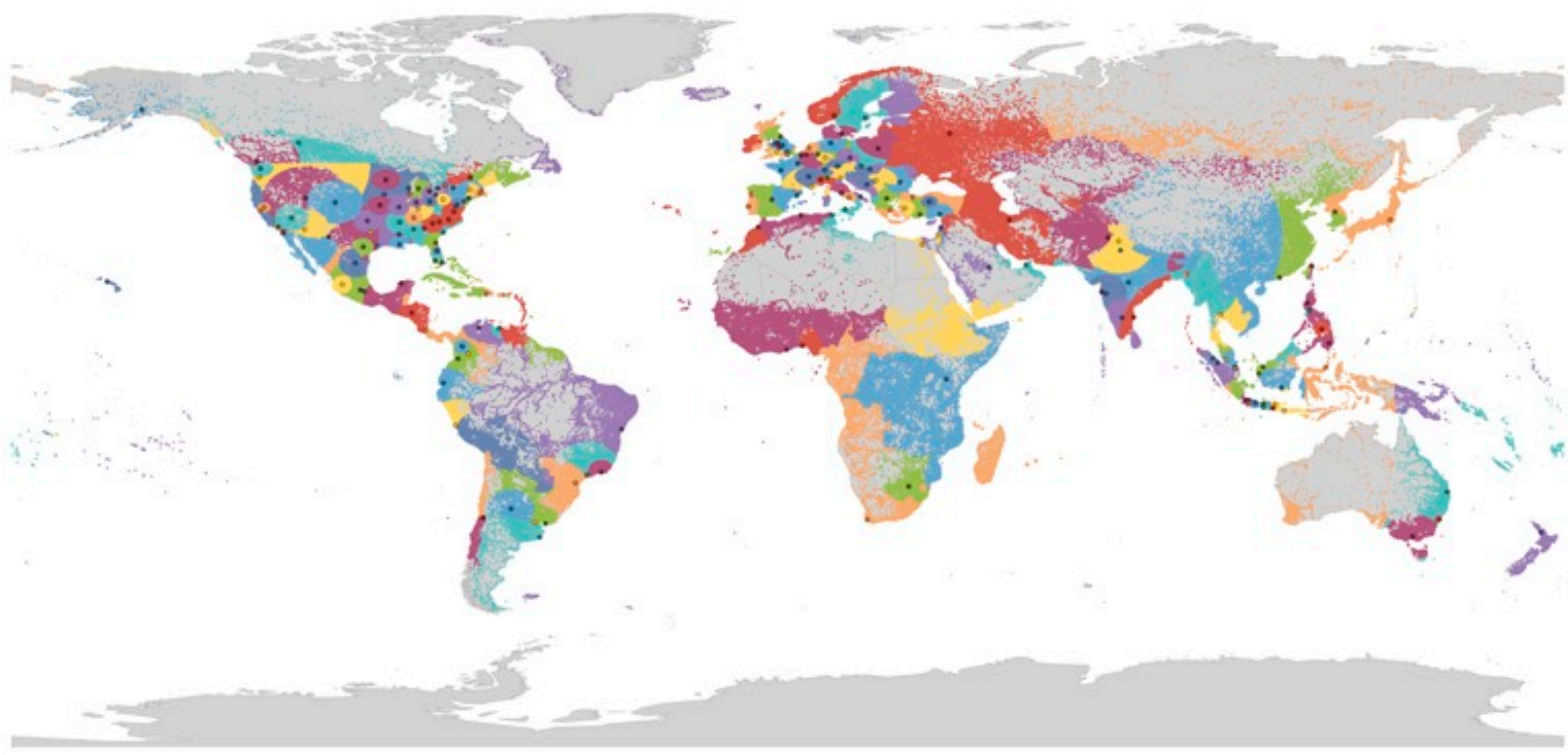
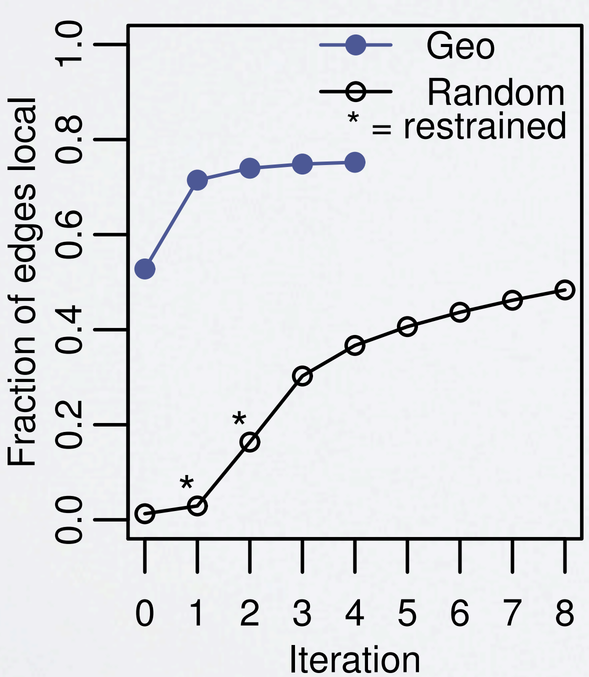


Results: Iteration convergence

- Geographic initialization ‘converges’ within **1 step**

Facebook (n=800m, |E|=68b)



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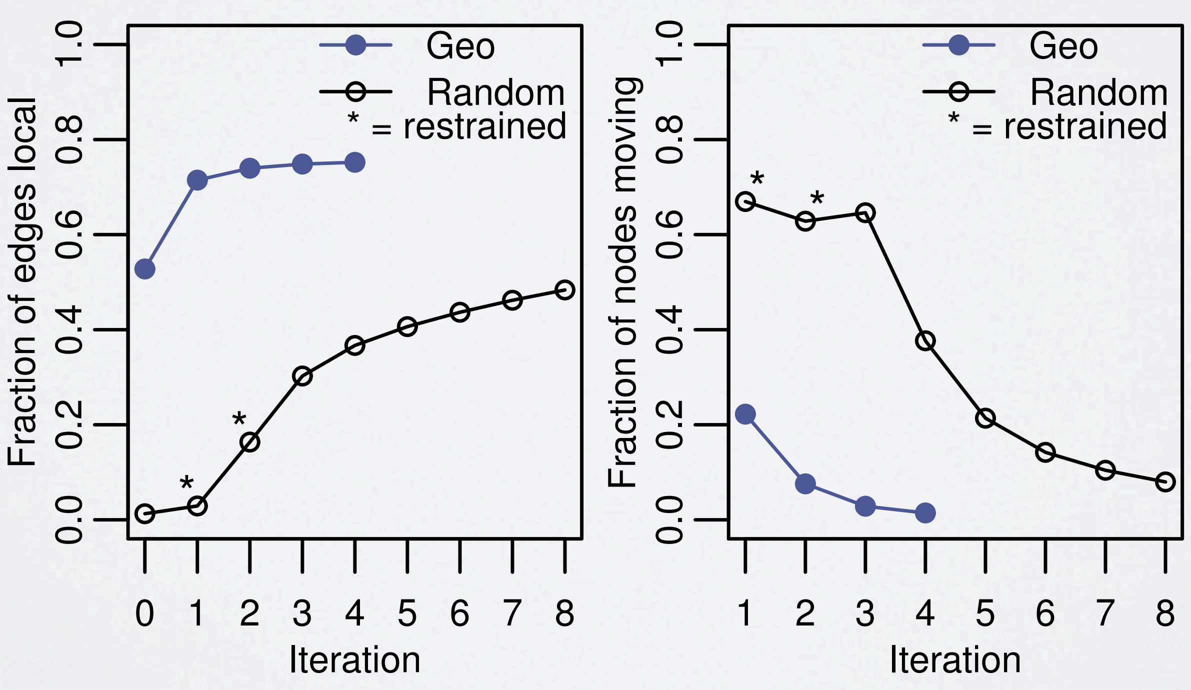

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  Use ‘**restraint**’: only move big gainers (*s below)
- LJ partitioning quality not so dependent on # partitions:
  BLP exploiting primarily **local structure.**

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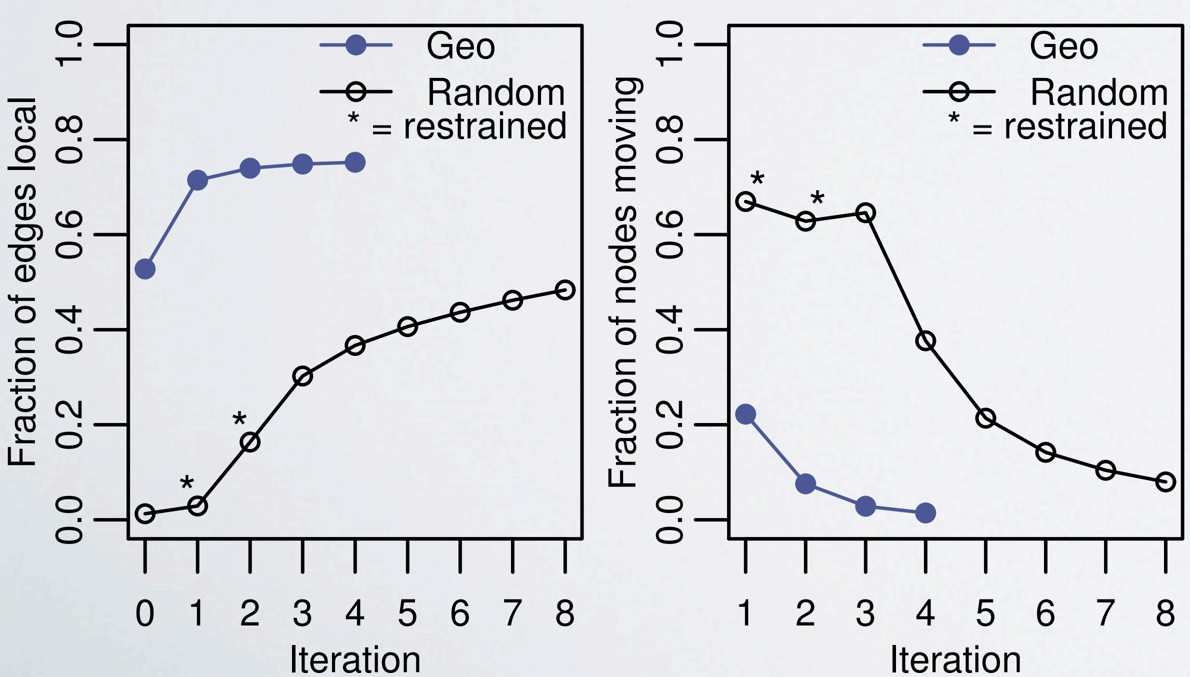
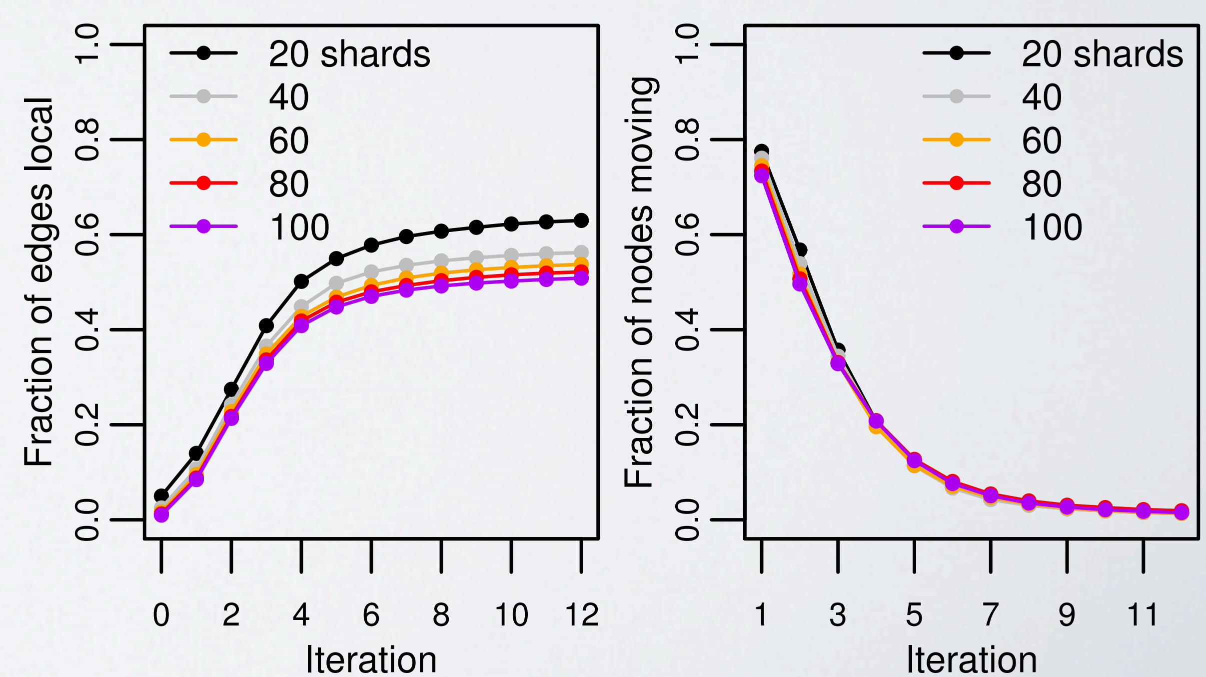
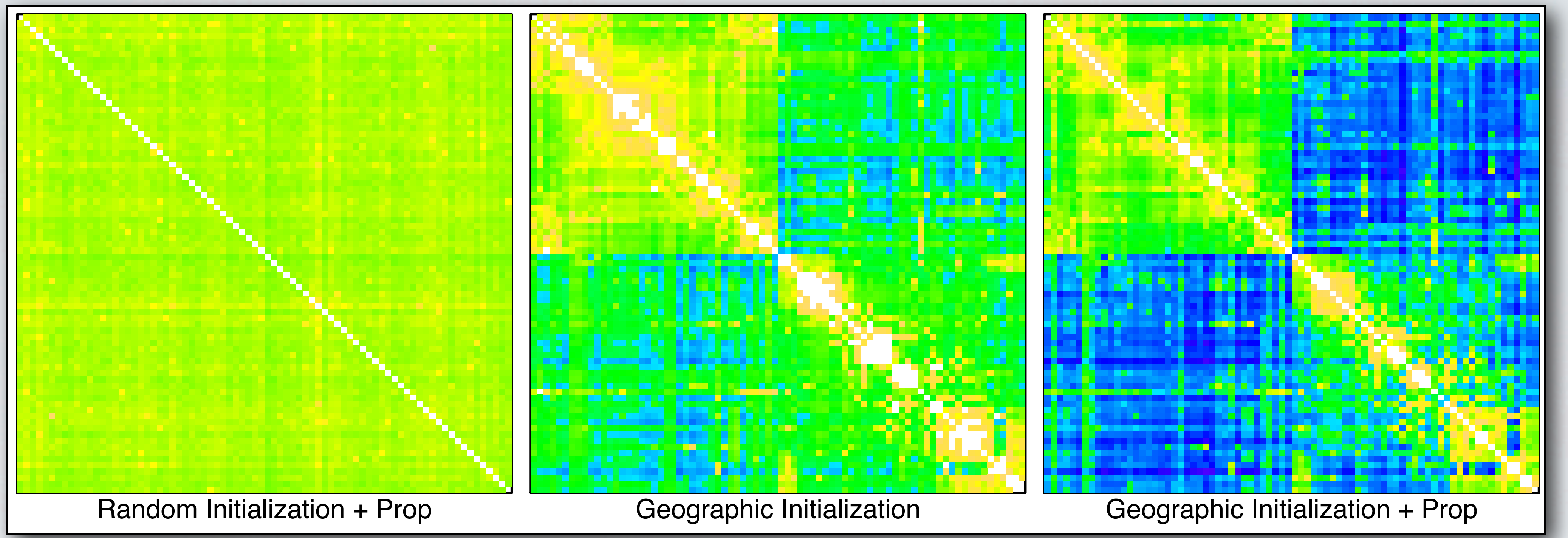


LiveJournal (n=4.8m, |E|=42.8m)

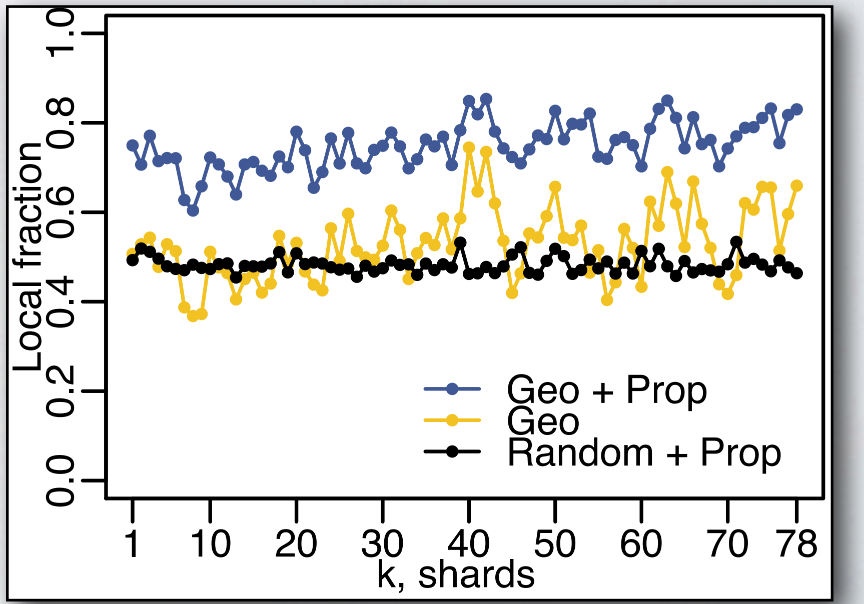


Results: Machine adjacency matrix



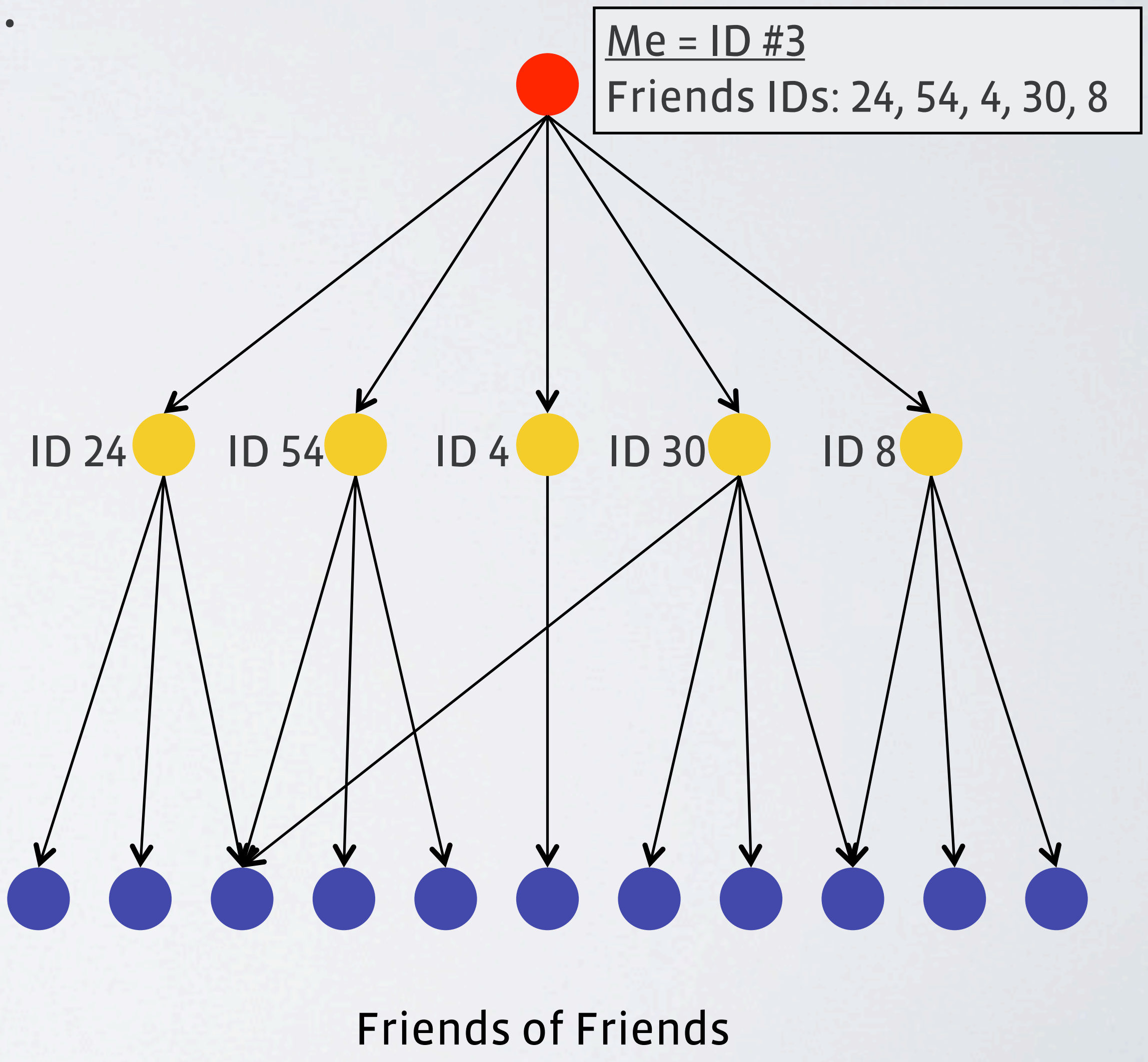
- Random initialization + 8 step prop
- Geographic initialization ONLY
- Geographic + 1 step prop

- Targeting n=78 machines:
  2 racks of 39, visible as blocks



‘People You May Know’

- PYMK = ‘People You May Know’
- Ranked suggestion of friends-of-friends (FoFs) as friends.
- Average user has 40k FoFs, widely distributed.
- Ranks **145,000,000 suggestions per second**.
- Graph distributed across 78 machines with 72GB RAM each.



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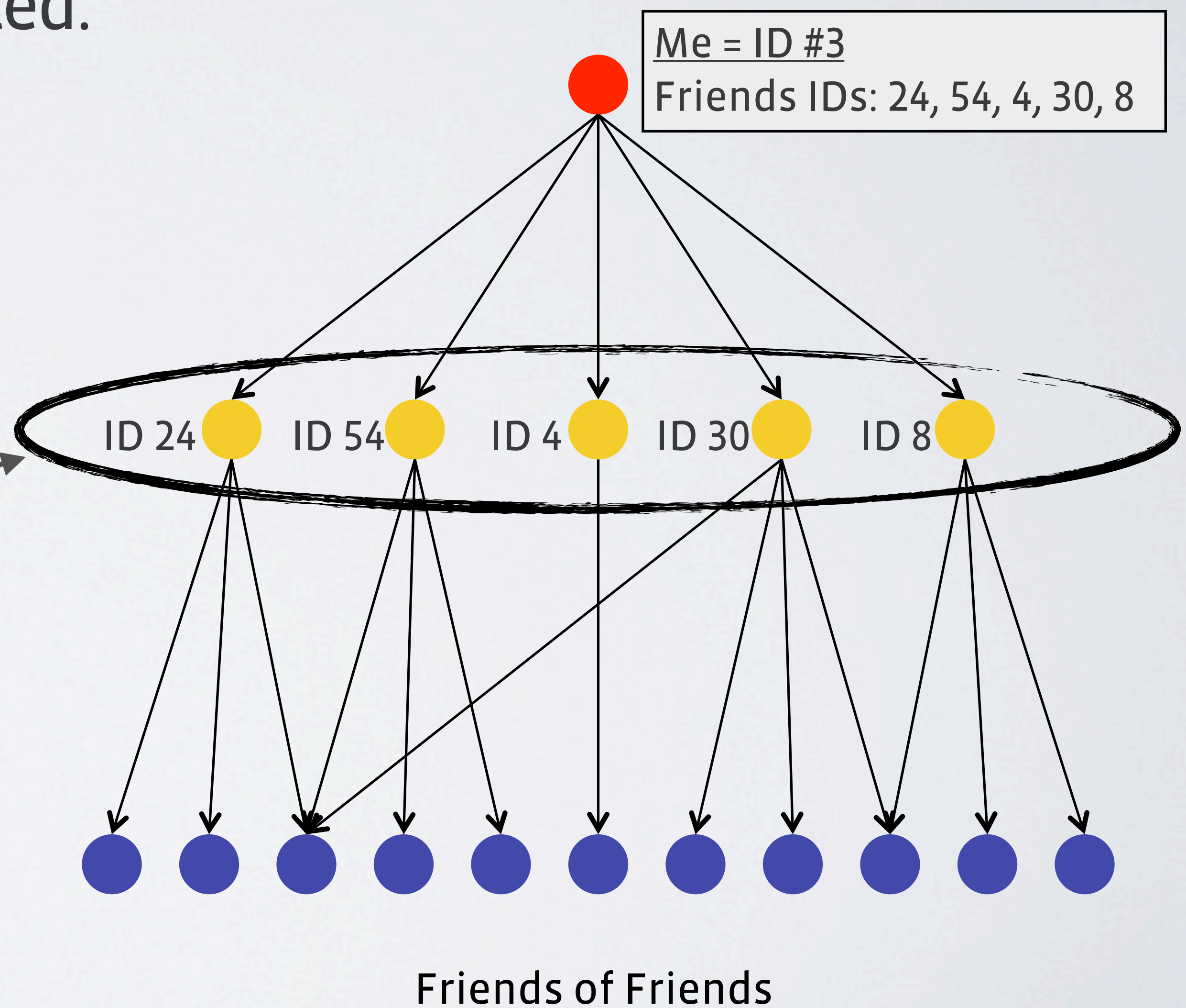
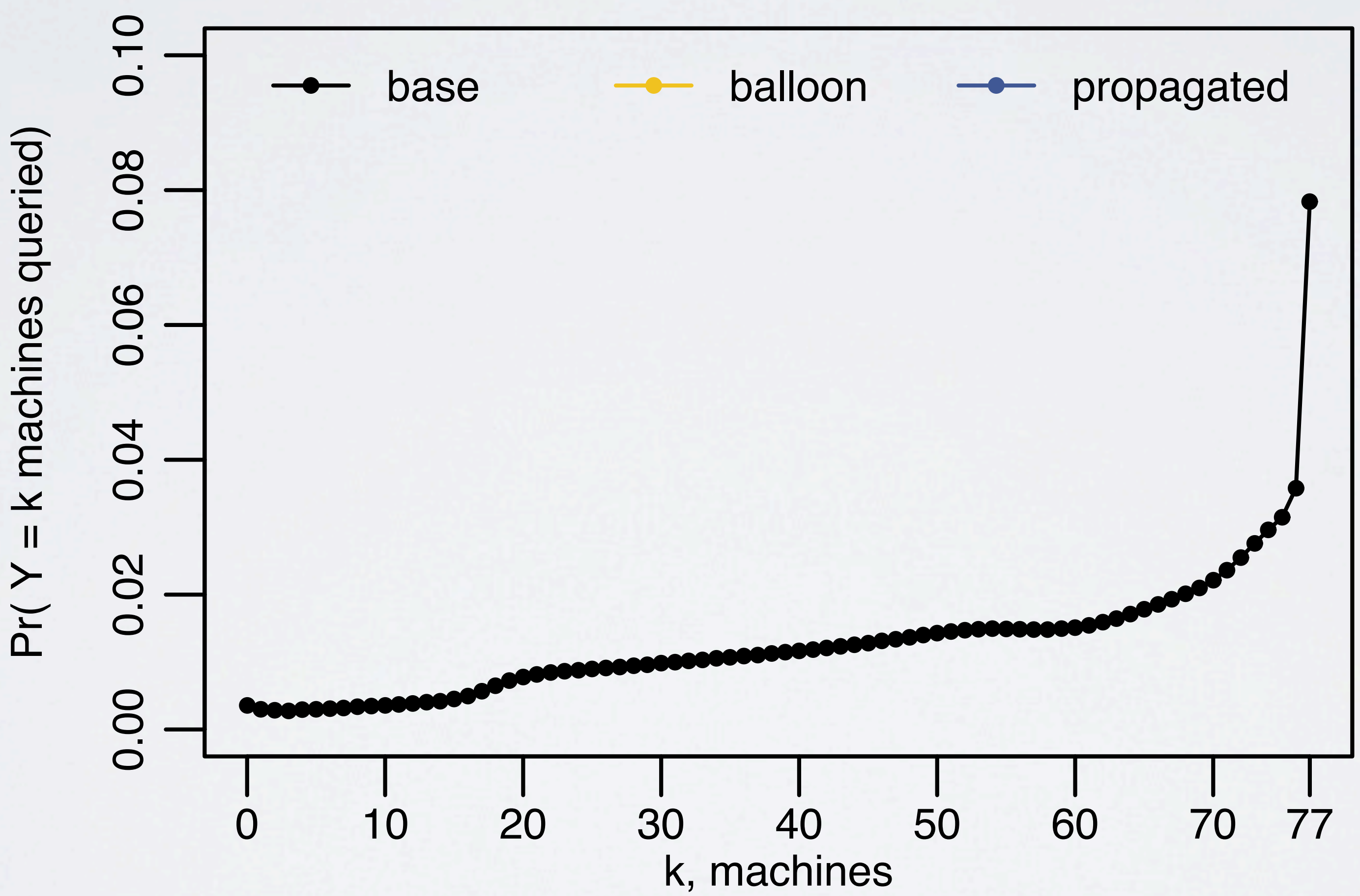
Want to shard so that
my friends on same machine as me!



Results: PYMK request concentration

- Median number of machines hit per query reduced from **60** to ?.



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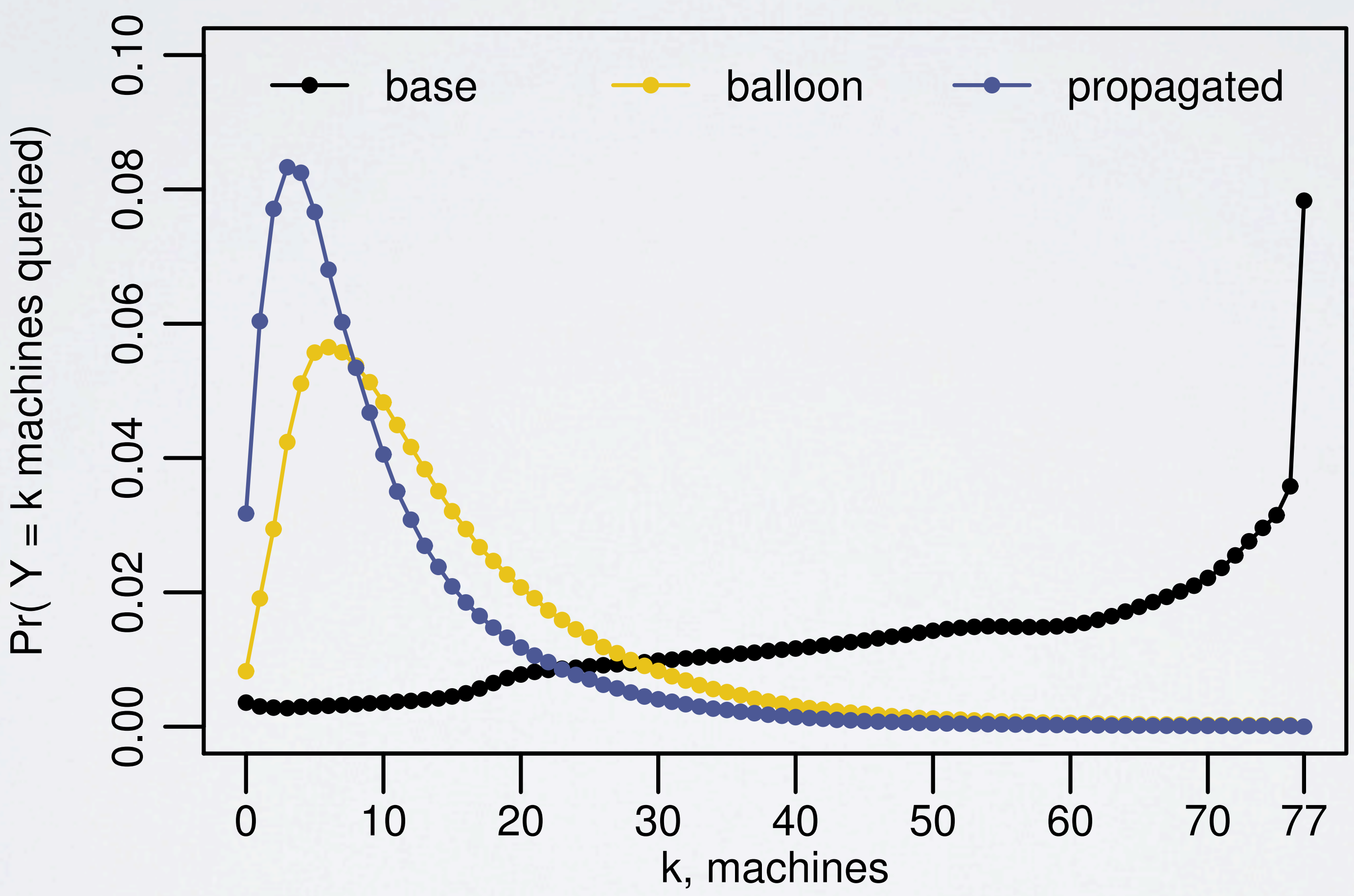

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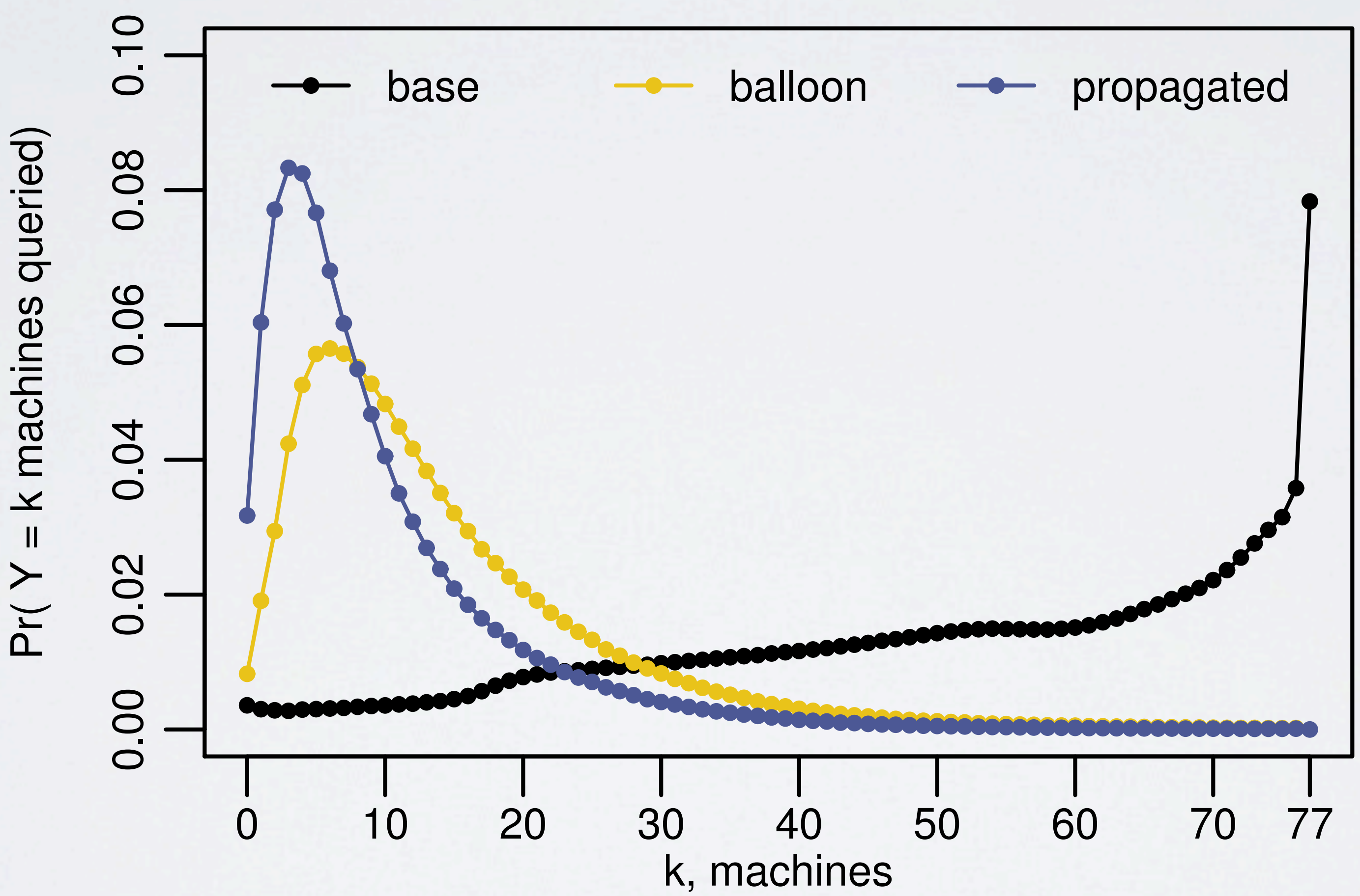
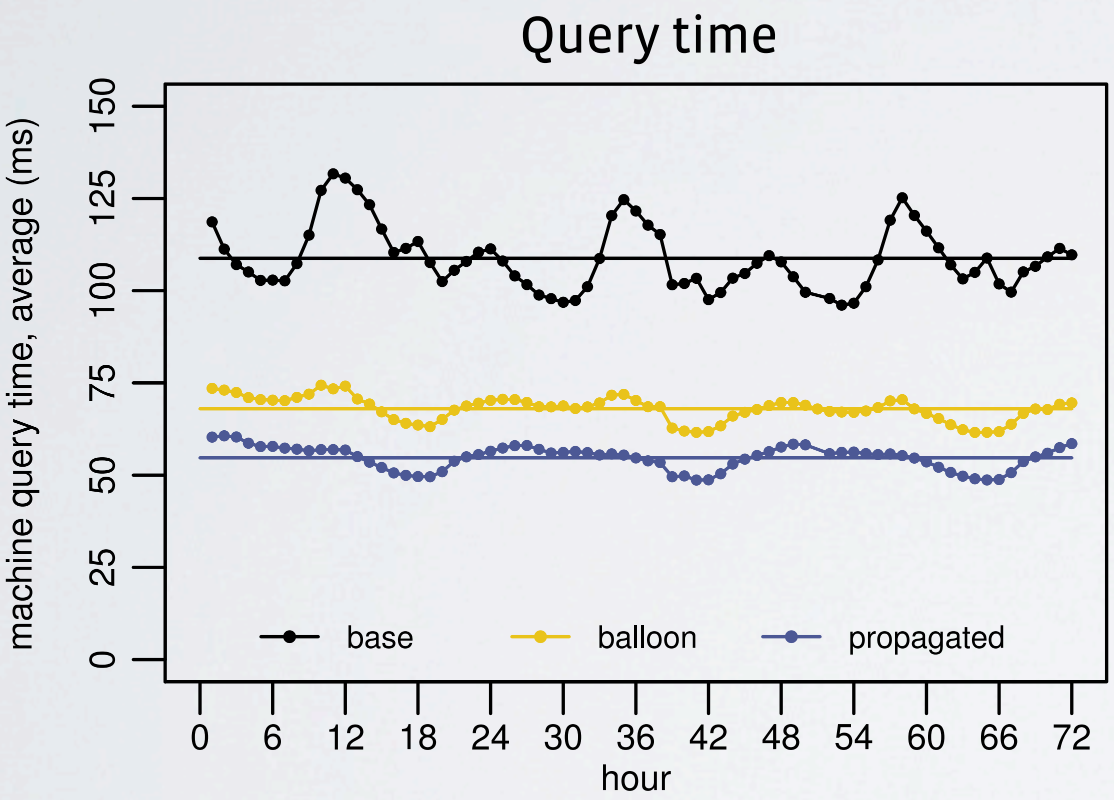
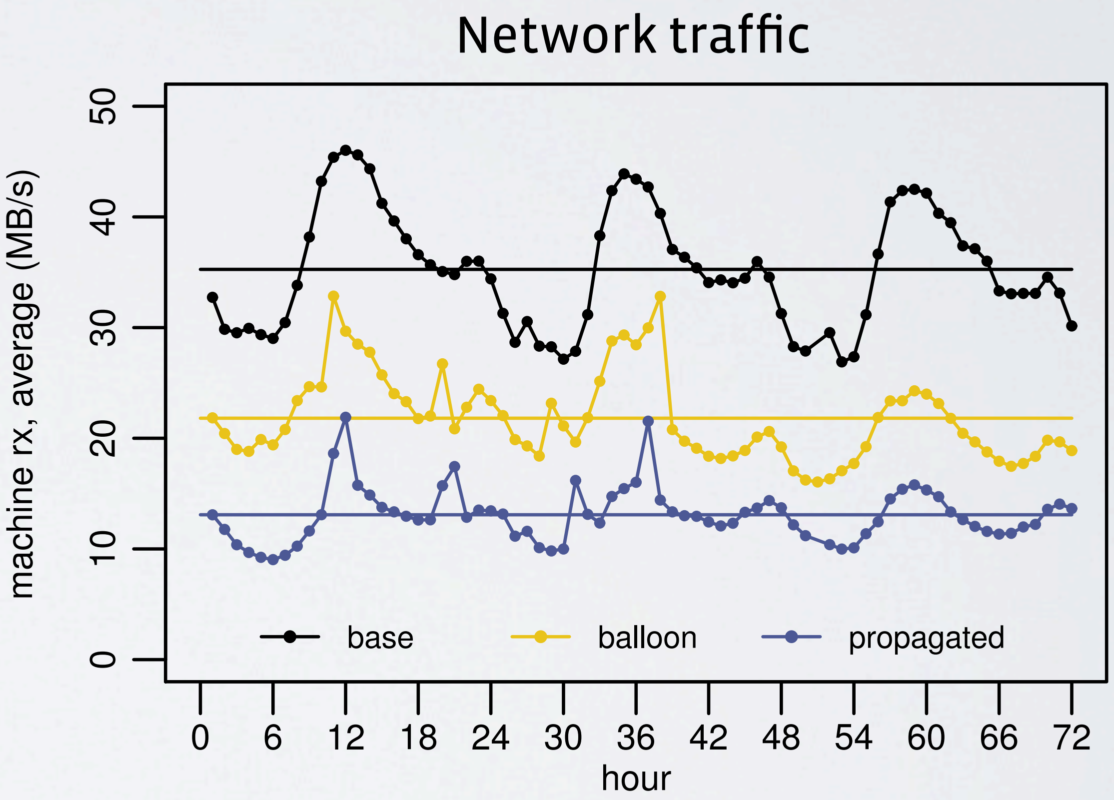


- **Query time**: What about overhead? Faster or slower?

Results: Query time / network traffic

- Median number of machines hit per query reduced from **60** to **9**.
- **Query time** reduced by **49%**, traffic reduced by **63%**:





Conclusions and Future work

- Label propagation is fast, we show it can be constrained
- Social networks very clustered, making local algorithms very effective
- Geographic metadata very useful

- Sharding greatly improves distributed graph computations such as PYMK